

Some Comments on “Split” Supersymmetry

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Abstract

An argument against tolerating finetuning in the Higgs sector is presented, by emphasizing the difference between (well understood) quantum corrections to scalar masses and the (unsolved) problem of the cosmological constant. I also point out that “split” supersymmetry, where all scalars except one Higgs boson have masses many orders of magnitude above the weak scale, is not compatible with simple mechanisms of transmitting supersymmetry breaking (gravity, gauge or anomaly mediation), unless a second, independent finetuning of parameters is introduced. This finetuning is required to obtain an acceptable ratio of vacuum expectation values $\tan\beta$.

Supersymmetry was originally considered something of a mathematical curiosity [1]. It began to be taken seriously as a realistic extension of the Standard Model (SM) of particle physics only after it was realized [2] that it can solve the finetuning problem of the scalar sector of the SM, by canceling all quadratically divergent corrections to the mass of the Higgs boson(s) required to break the electroweak gauge symmetry spontaneously. This is the primary motivation for introducing supersymmetry in our description of nature.

Of course, supersymmetry has to be broken; no generally accepted mechanism to achieve this has as yet emerged. Indeed, supersymmetry breaking is widely perceived to be the ugly side of supersymmetric extensions of the SM, since it can easily lead to problems with flavor changing neutral currents (FCNC) and CP violation. However, these problems are just as easily solved if supersymmetry breaking terms are sufficiently universal and real. In fact, it was shown [3] that in the minimal supersymmetric extension of the SM (MSSM), universal supersymmetry breaking terms generated at a high energy scale allow for radiative $SU(2) \times U(1)_Y \rightarrow U(1)_{\text{em}}$ symmetry breaking, thereby finding a dynamical explanation for the negative squared mass of the Higgs boson that has to be put “by hand” into the Lagrangian of the SM. This mechanism, as well as general finetuning arguments [4], indicate that superparticle masses should not lie much above the weak scale, leading to good prospects for their discovery at future high-energy colliders such as the LHC [5]. Moreover, radiative $SU(2) \times U(1)_Y$ breaking works best for large top mass. Although it can also be realized with moderate top mass [6], the fact that the top quark is the heaviest known elementary particle can therefore be counted as argument in favor of softly broken weak-scale supersymmetry [7].

A little later it was realized [8] that broken supersymmetry also provides a good candidate for the Cold Dark Matter (CDM) which we now believe to form some 85% of all matter in the universe, and to contribute about 25% of its energy density [9]. Finally, measurements at LEP showed conclusively [10] that in the SM the gauge couplings do not unify at a point, whereas at the MSSM unification at the percent level can easily be achieved.* Recently it has been pointed out [11] that these secondary virtues of the MSSM (and similar supersymmetric models) are shared by models where almost all scalars have very large masses, possibly even of order of the unification scale. Since the superpartners of the fermions come in complete representations of $SU(5)$, they have no impact (to one-loop order) on whether or not the gauge couplings unify. As long as the gauginos and higgsinos are kept light, unification will work more or less like in the MSSM. Moreover, the lightest neutralino can still make a good CDM candidate, so long as it is not Bino-like. Of course, one Higgs doublet has to be kept light in order to achieve electroweak symmetry breaking. The authors of ref.[11] coined the phrase “split supersymmetry” for this kind of model.

These models have attracted a fair amount of interest although they make no pretense of solving the finetuning problem of the SM, i.e. they abandon the primary virtue of supersymmetry. The argument given in [11] essentially amounts to the statement that finetuning anyway seems to be required to solve the cosmological constant problem, so one might as well allow finetuning also in other sectors of the theory. Indeed, the “string landscape” is supposed to take care of this little thing for us.

However, this argument brushes over the fact that there is an essential difference between the cosmological constant and the mass of the Higgs boson. The former is a macroscopic quantity which can be defined only in the framework of a theory of gravity, presumably General Relativity or an extension thereof. An understanding of the connection between the

*Exact unification is not expected in the presence of unknown high-scale threshold corrections.

vacuum energy predicted by quantum field theory and the cosmological constant may therefore only be possible in the framework of a quantum theory of gravity. In contrast, the mass of the Higgs boson is a microscopic quantity, which should be computable using the well-known rules of quantum field theory. It therefore seems premature, to say the least, to interpret our lack of understanding of the cosmological constant (which is a very serious theoretical problem indeed!) as *carte blanche* for allowing finetuning anywhere in our description of nature.

Moreover, realistic models with “split” supersymmetry are less easy to construct than their recent popularity suggests. The reason is that one needs vacuum expectation values (vevs) for the neutral components of *both* Higgs doublets, $v_1 = \langle H_d^0 \rangle$ and $v_2 = \langle H_u^0 \rangle$, in order to give masses to both the bottom and the top quark. If we want to keep the Yukawa couplings in the perturbative domain, which is suggested for a perturbative unification of the gauge couplings, the parameter $\tan \beta \equiv v_2/v_1$ should lie in the range

$$0.5 \lesssim \tan \beta \lesssim 100. \quad (1)$$

This quantity can be calculated by minimizing the (tree-level) Higgs potential [12], which can be written as

$$V_{\text{Higgs}} = m_{H_u}^2 |H_u^0|^2 + m_{H_d}^2 |H_d^0|^2 - (B\mu H_u^0 H_d^0 + h.c.) + \frac{g^2 + g_Y^2}{8} \left(|H_u^0|^2 - |H_d^0|^2 \right)^2. \quad (2)$$

Here g and g_Y are the $SU(2)$ and $U(1)_Y$ gauge couplings, respectively, and μ is the mass parameter coupling the two Higgs superfields in the superpotential. Minimization of (2) gives [12]

$$\sin 2\beta = \frac{2B\mu}{m_{H_u}^2 + m_{H_d}^2}. \quad (3)$$

In models with “split” supersymmetry, one has $m_{H_u}^2 \sim \mathcal{O}(m_{\text{weak}}^2)$, $m_{H_d}^2 \sim \mathcal{O}(m_{\text{SUSY}}^2) \gg |\mu|^2 \sim \mathcal{O}(m_{\text{weak}}^2)$. All other scalars also have masses $\mathcal{O}(m_{\text{SUSY}})$, leading to finetuning at the level $(m_{\text{weak}}/m_{\text{SUSY}})^2$.

Recall that in “split” supersymmetry, gaugino masses are assumed to be $\mathcal{O}(m_{\text{weak}})$. This could be achieved by a softly broken R symmetry. However, an R symmetry that allows a supersymmetric μ term would forbid a nonvanishing B , so that $|B| \sim \mathcal{O}(m_{\text{weak}})$, leading to $\tan \beta \sim \mathcal{O}(m_{\text{SUSY}}^2/m_{\text{weak}}^2)$, and hence

$$m_{\text{SUSY}} \lesssim 10 m_{\text{weak}} \quad \text{if } |B| \sim \mathcal{O}(m_{\text{weak}}), \quad (4)$$

well below the scales usually considered in “split” supersymmetry.

One thus has to choose the R -charges such that they forbid μ as well as the gaugino masses, but allow $B\mu$ to be nonzero. One then faces the challenge to generate μ (which conserves supersymmetry but breaks this R -symmetry) and gaugino masses (which break both supersymmetry and this R -symmetry) of roughly the same size. Note that the well-known supergravity solutions [13] of the “ μ -problem” will not work here. Let us nevertheless assume that μ and gaugino masses of the appropriate size have been produced, and investigate the size of $|B\mu|$ predicted by simple mechanisms of transmitting supersymmetry breaking to the visible sector.

If supersymmetry breaking is transmitted by gravitational-strength interactions to the visible sector (which automatically happens once supersymmetry is embedded in a supergravity

theory [4]), one expects $|B| \sim \mathcal{O}(m_{\text{SUSY}})$, leading to $\tan\beta \sim \mathcal{O}(m_{\text{SUSY}}/m_{\text{weak}})$. This will be compatible with the constraint (1) only if

$$m_{\text{SUSY}} \lesssim 100 m_{\text{weak}} \quad [\text{gravity mediation : } |B| \sim \mathcal{O}(m_{\text{SUSY}})]. \quad (5)$$

This is sufficient to solve all potential problems with FCNC and CP violation, but leads to a spectrum more reminiscent of so-called “inverted hierarchy” or “more minimal supersymmetry” models [14], rather than what is typically considered for “split” supersymmetry.

How can the bound (5) be circumvented? First of all, the upper bound in (1) has been obtained by requiring that $m_b = \lambda_b \langle H_d^0 \rangle$ be sufficiently large for a Yukawa coupling $\lambda_b \lesssim \mathcal{O}(1)$. Note that the Higgs potential should be minimized at scale m_{SUSY} , which by assumption is exponentially larger than m_{weak} . However, below m_{SUSY} the b -quark mass runs as in the SM, i.e. only increases by a factor $\lesssim 2$ when going down to scale m_t . This effect has already been included by allowing $\tan\beta$ to be as large as 100.

The limit (5) can thus only be evaded if $|B| \gg m_{\text{SUSY}}$. In fact, this is easily possible in models with gauge mediated supersymmetry breaking (GMSB) [15], where the $B\mu$ term can be generated already at one-loop level, whereas scalar masses are only generated at two-loop level.[†] This then allows $|B\mu| \sim 100 m_{\text{SUSY}}$, leading to

$$m_{\text{SUSY}} \lesssim 10^4 m_{\text{weak}} \quad [\text{GMSB : } |B\mu| \sim 100 m_{\text{SUSY}}]. \quad (6)$$

This indeed allows a considerable “splitting” of the superparticle spectrum. However, m_{SUSY} would then still have to be many orders of magnitude below the scale of Grand Unification, and even well below most other “intermediate” scales (e.g. the Peccei–Quinn scale associated with the spontaneous breaking of a possible $U(1)$ symmetry that can be used to rotate away the QCD θ -term [16]). Moreover, gluinos, while sufficiently long-lived to have detectable decay lengths, would hardly be “meta-stable”, which is supposed to be a hallmark of models with “split” supersymmetry.

The third simple mechanism to transmit supersymmetry breaking to the visible sector goes under the name of anomaly mediation [17]. It naturally gives (sufficiently) flavor-universal soft scalar masses not to suffer from FCNC problems. This mechanism always produces gaugino and sfermion masses of the same order of magnitude; it can therefore not lead to a “split” supersymmetry spectrum.

How could even larger values of $|B|$ be generated? Replacing μ by the vev of a visible sector singlet field N does not help. In that case $|B| = |A\langle N \rangle|$, where A is a *trilinear* soft breaking parameter, which contributes to electroweak gaugino masses through finite one-loop diagrams, and to all gaugino masses (via the A -parameters associated with squarks and sleptons) through two-loop renormalization group equations. The requirement that gaugino masses are roughly of order of the weak scale would then give a bound on m_{SUSY} which is stronger than that in (6).

The only known way to produce the required $|B| \gg m_{\text{SUSY}}$ relies on D -term supersymmetry breaking [18] with direct (tree-level) coupling to the visible sector. Of course, this has been attempted in the early days of supersymmetry phenomenology [4]; it had been largely abandoned since it is difficult to find non-anomalous models where all squared sfermion masses are positive. Acceptable models tend to be quite baroque; for example, the model of ref.[18] needs six SM singlets, plus a tripling of the MSSM matter fields by introducing vector-like

[†]In fact, this is a problem for GMSB models with the natural choice $m_{\text{SUSY}} \sim m_{\text{weak}}$; somewhat complicated constructions are required to suppress the 1-loop contribution to $B\mu$ [15].

partners for each MSSM matter superfield.[‡] A very recent model [19] based on an anomalous $U(1)$ requires an additional strongly interacting sector (i.e. an additional scale in the theory), and leads to an only moderately split spectrum, with sfermions not much above the bound (6).

In the theoretically well motivated, (comparatively) simple scenarios with gravity- or gauge-mediated supersymmetry breaking the bounds (5), (6) can only be evaded if one tunes $m_{H_d}^2 \ll m_{\text{SUSY}}^2$. For example, $|B| \sim \mathcal{O}(m_{\text{SUSY}})$ would require $m_{H_d}^2 \lesssim \mathcal{O}(100 m_{\text{weak}} m_{\text{SUSY}})$. This would require a *second, independent* finetuning if m_{SUSY} is above the range (5), over and above the finetuning required to keep $m_{H_u}^2$ of order m_{weak}^2 .

In summary, I have argued that the (unsolved) problem of the cosmological constant should not be interpreted as evidence in favor of finetuning in other sectors of the theory. Moreover, a very large splitting between the weak scale and the scale of sfermion masses can be achieved in simple, well-motivated models of supersymmetry breaking only if the mass of the second Higgs doublet is intermediate between these scales, i.e. well below the masses of the sfermions. Imposing such a hierarchy between the masses of heavy scalars would require a second, independent finetuning, in addition to that needed to have $m_{\text{weak}} \ll m_{\text{SUSY}}$. This should be of concern in any probabilistic (e.g. “string landscape”) interpretation of this kind of model.

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